Stochastic models of innovation processes

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1. Introduction

- Stochast. effects play important role in biological and socioeconomical processes,
- examples: innovations and technology transfer,
- the simple picture: the new is the better and replaces the bad old is not always true !!!
- Role of chance, of stochastic effects!
- We consider two simple math models:

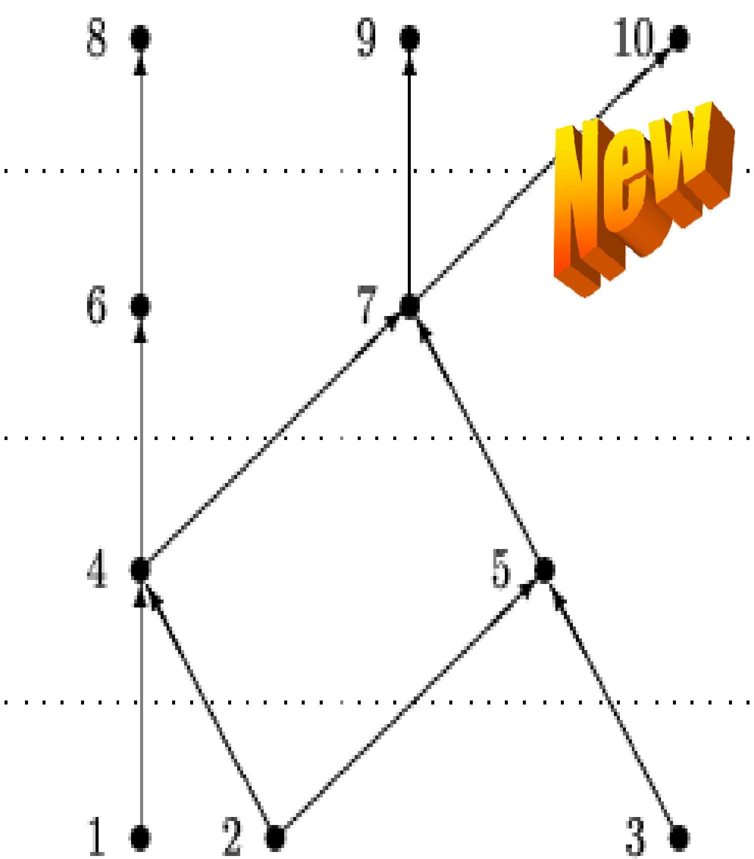
1) Discrete Urn-model: what happens if new technologies appear on the market, result of competition

- stochastic effects are important if the advantage of the NEW is small
- selection is vague with a broad region of neutrality; in order to win the competition the NEW needs big advantage.
- technologies with nonlinear growth rates have only a chance to win in niches or with external support.

2) Models based on continuous Brownian dynamics: Transitions to other technologies

- Technologies are modelled as active Brownian particles with velocity-dependent friction, collective interactions and external confinement.
- We simulate the dynamics of such transitions by Langevin equations and estimate the transition rates.

2. Stochastic Urn Model



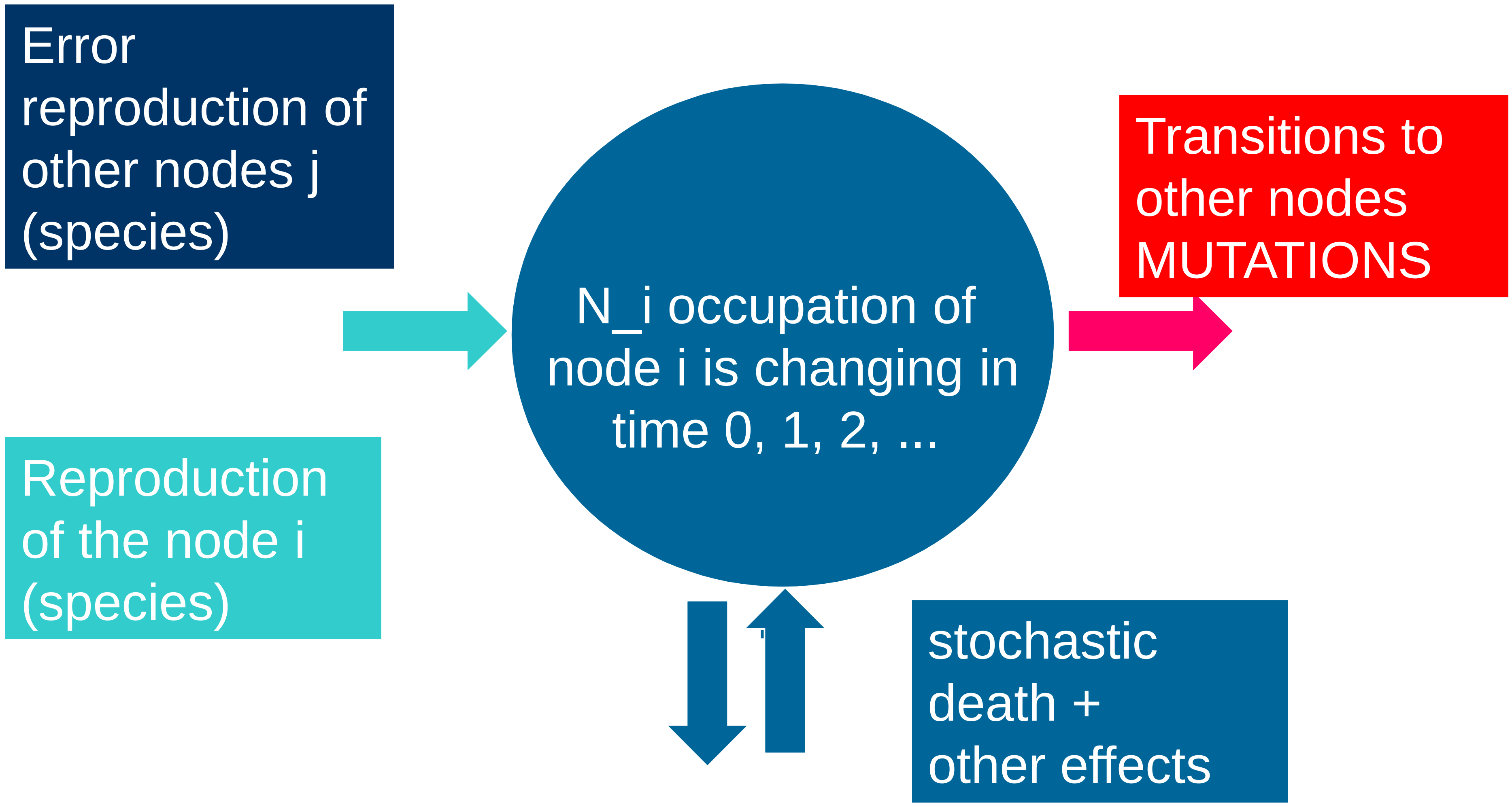
- Evolution as dynamics in a network
- A special role play transitions to new technologies (node 10).
- By changing the old, by new ideas, inventions =formally a transition to a new node
- Fate of the NEW = stochastics on nodes
- Urn models !!!

On history of stochastic urn models

- Paul & Tatjana Ehrenfest 1907: Urn models (flees jump from dog to dog) . First biophys. Appl.!!!
- Bartholomay 1958/59, Bartlett 1960: Birth and death processes, survival probabilities
- Kimura/Eigen: Applications to problems of evolution

Applications to genetics population dynamics, etc.

Stochastic change in occupation of nodes

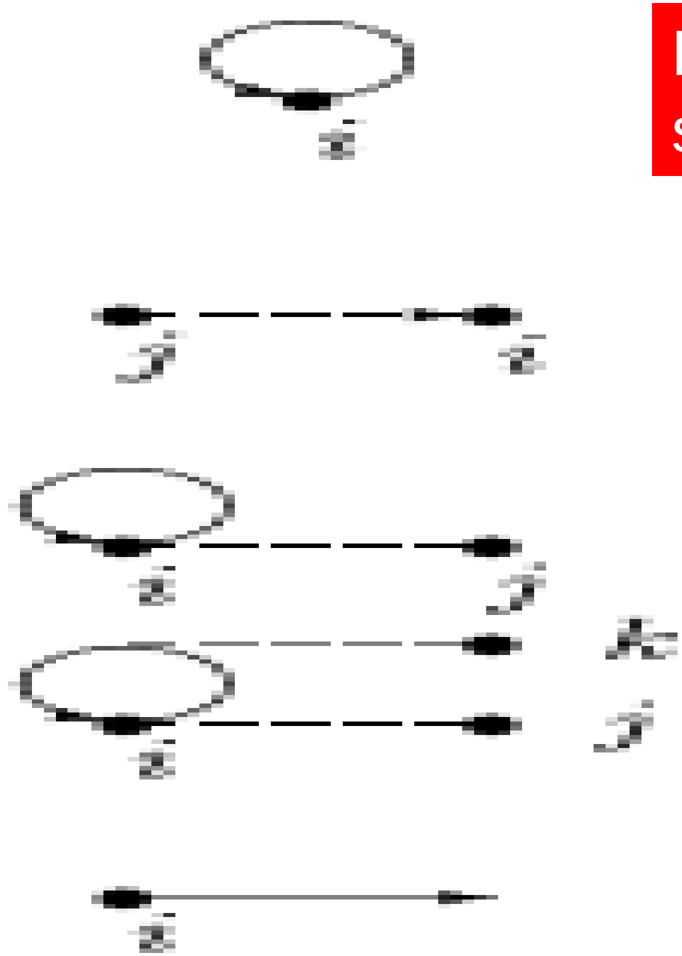


Transformation of given d.e. of Volterra type to stochastic models. Recipe is clear only for polynoms (transition probs ~ coeff.)

Special cases: Lotka-Volterra, Eigen-Schuster,..

$$\frac{d}{dt}x_i = \sum_j \left(A_{ij}\, x_j + B_{ij}\, N x_i x_j + \sum_k C_{ijk}\, N^2 x_i x_j x_k \right)$$

Network: Use edges between the nodes for characterizing processes like self-reproduction, mutations, catalytic reproductions, decay etc.

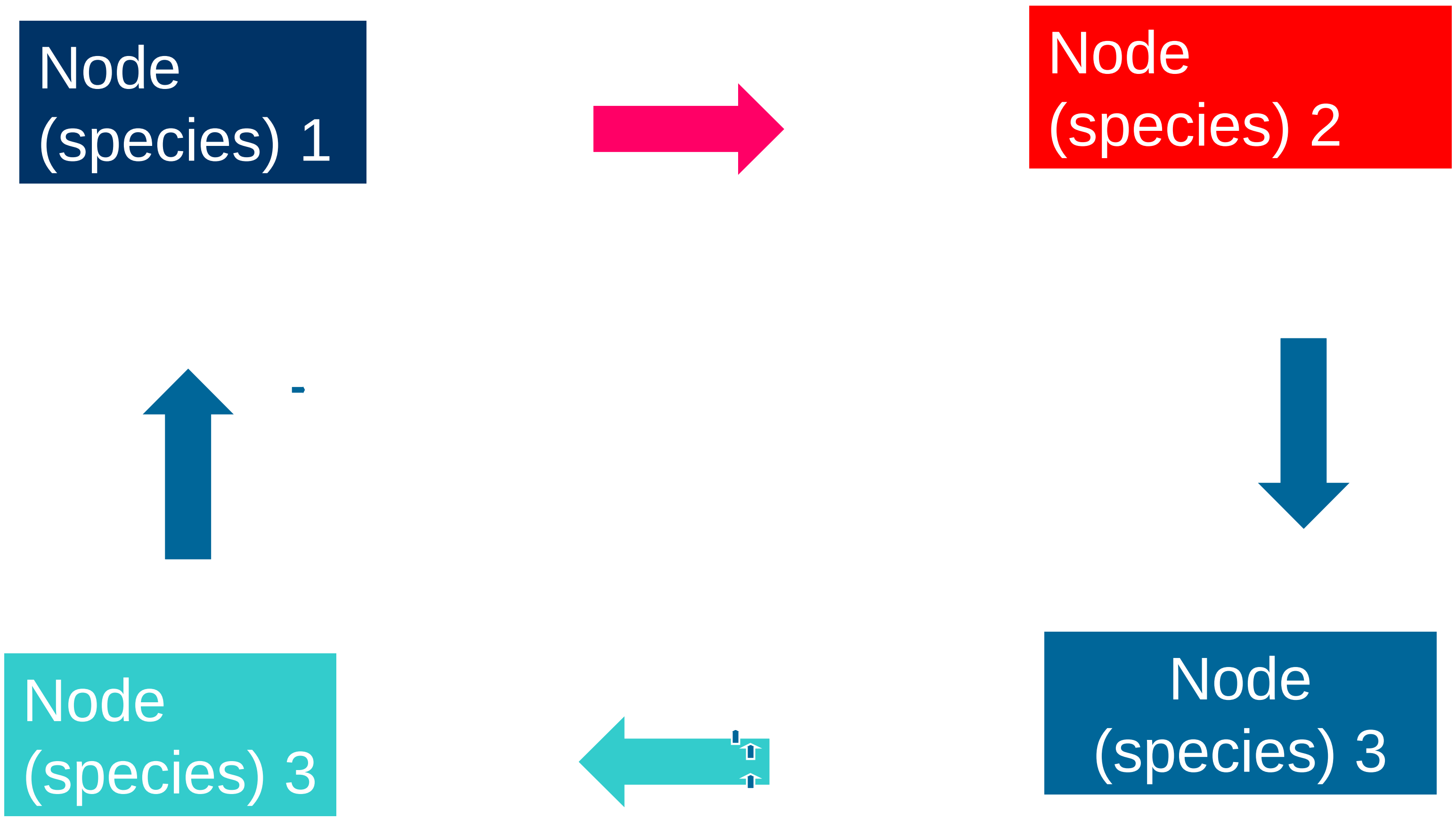


Loop = selfreproduction

When we need stochastic analysis ?

- As a rule stochast effects are small since (N >> 1). However there are other cases (N=0,1): Innovations!
- Of special interest innov with hypercycle charactkter (see theory of HC by Eigen/Schuster)
- HC are ring nets of species/ technologies with hyperbolic growth (WINDOWS, GOOGLE, all or nothing)

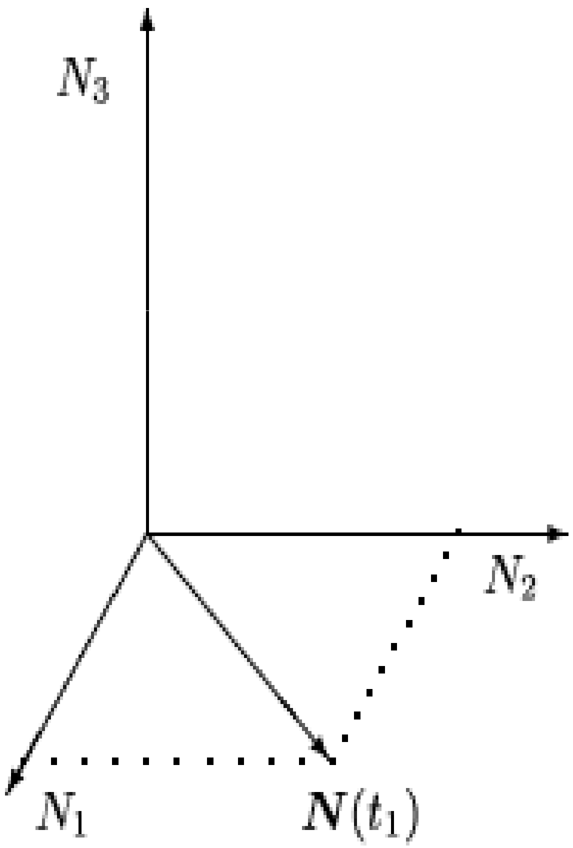
Hypercycles of technology nets

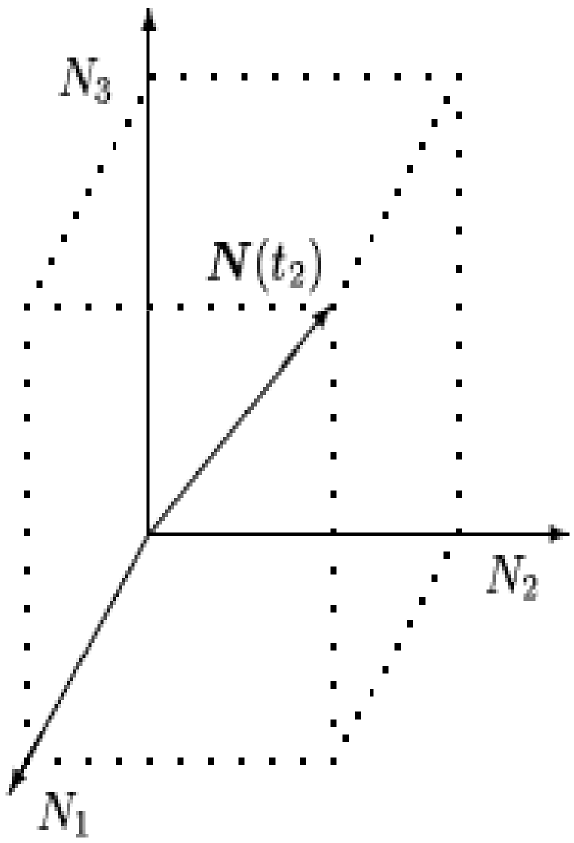


Stochastic models (birth& death): define nodes for species and occupation numbers

$\circ_i$	non-occupied	$N_i = 0$
$\bullet_i$	occupied	$N_i > 0$

Occupation number space

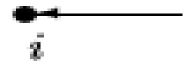
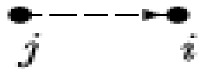
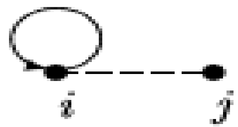
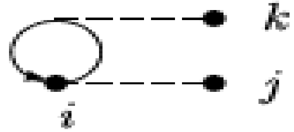
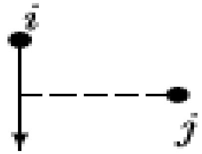
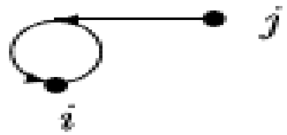
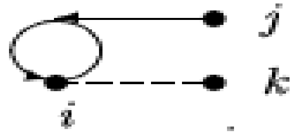




Def transition probs dep on coefficients

$$W(N_1,\ldots,N_i+1,\ldots,N_j-1,\ldots,N_s | N_1,\ldots,N_i,\ldots,N_j,\ldots,N_s)$$

$$= A_{ij}N_j + B_{ij}N_iN_j + \sum_k C_{ijk}N_iN_jN_k$$

1. Spontaneous generation (simple innovation)	$A_i^{(0)}$	
2. Self-reproduction	$A_i^{(1)} N_i$	
Error reproduction	$A_{ij}^{(1)} N_j$	
Catalytic self-reproduction (sponsored self-reproduction)	$B_{ij}^{(1)} N_i N_j$	
	$C_{ijk}^{(1)} N_i N_j N_k$	
3. Spontaneous decay	$A_i^{(2)} N_i$	
Catalytic decay	$B_{ij}^{(2)} N_i N_j$	
4. Mutation (innovation)	$A_{ij}^{(3)} N_j$	
Mutation (innovation) with reproduction	$B_{ij}^{(3)} N_i N_j$	
	$C_{ijk}^{(3)} N_i N_j N_k$	

Formulate a master eq as balance of elementary processes, simplex cond N = const

$$\frac{\partial P(N;t)}{\partial t} = W(N|N')\,P(N') - W(N'|N)\,P(N)$$

$$N = \{N_0, N_1, N_2, \ldots, N_s\} .$$

How sharp is stochastic selection? What is stochastic neutrality ?

Node (species) 1

Node (species) 2

The message is:

Stochastic selection is very weak, nearly always neutral

Study binary competition : 1=OLD, 2=NEW

- Consider a two-component system:
- The MASTER with dominant occupation:
- The NEW species with one, or a few, representatives which try to survive and (if possible) to win the competition.
- In general we will assume that e NEW is better with respect to reproductive rates

Binary competition
$N_1 + N_2 = N = const$

$$\frac{\partial}{\partial t} P(N_1, N_2; t) = W^{+}_{N_2-1}(N_2|N_2-1)\ P(N_2-1;t)$$

$$+ W^{-}_{N_2+1}(N_2|N_2+1)\ P(N_2+1;t)$$

$$- W^{+}_{N_2}(N_2+1|N_2)\ P(N_2;t)$$

$$- W^{-}_{N_2}(N_2-1|N_2)\ P(N_2;t)$$

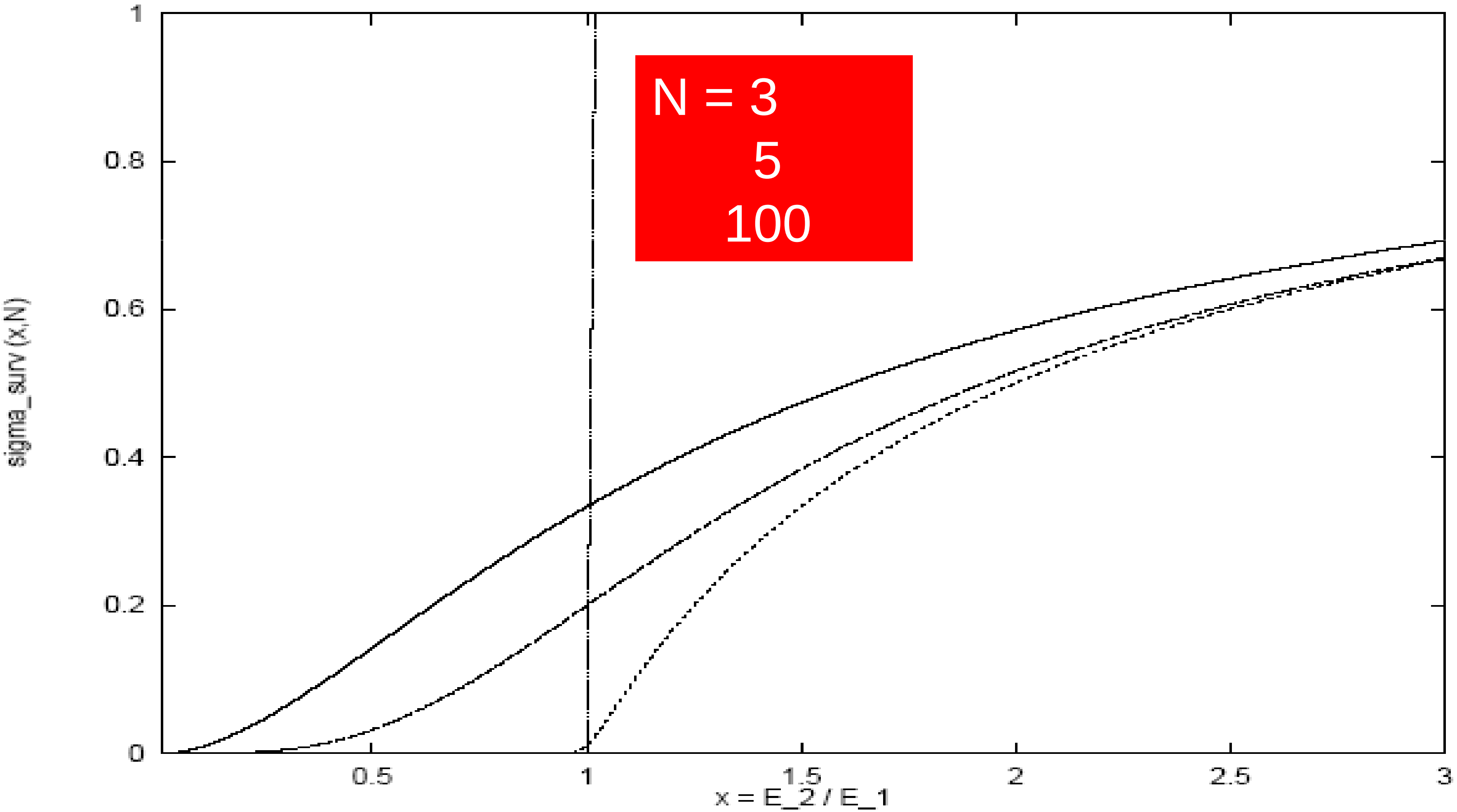
Only 1 independent variable N_2 (represent of the NEW)

Linear rates, prob of survival (Bartholomay, Bartlett)

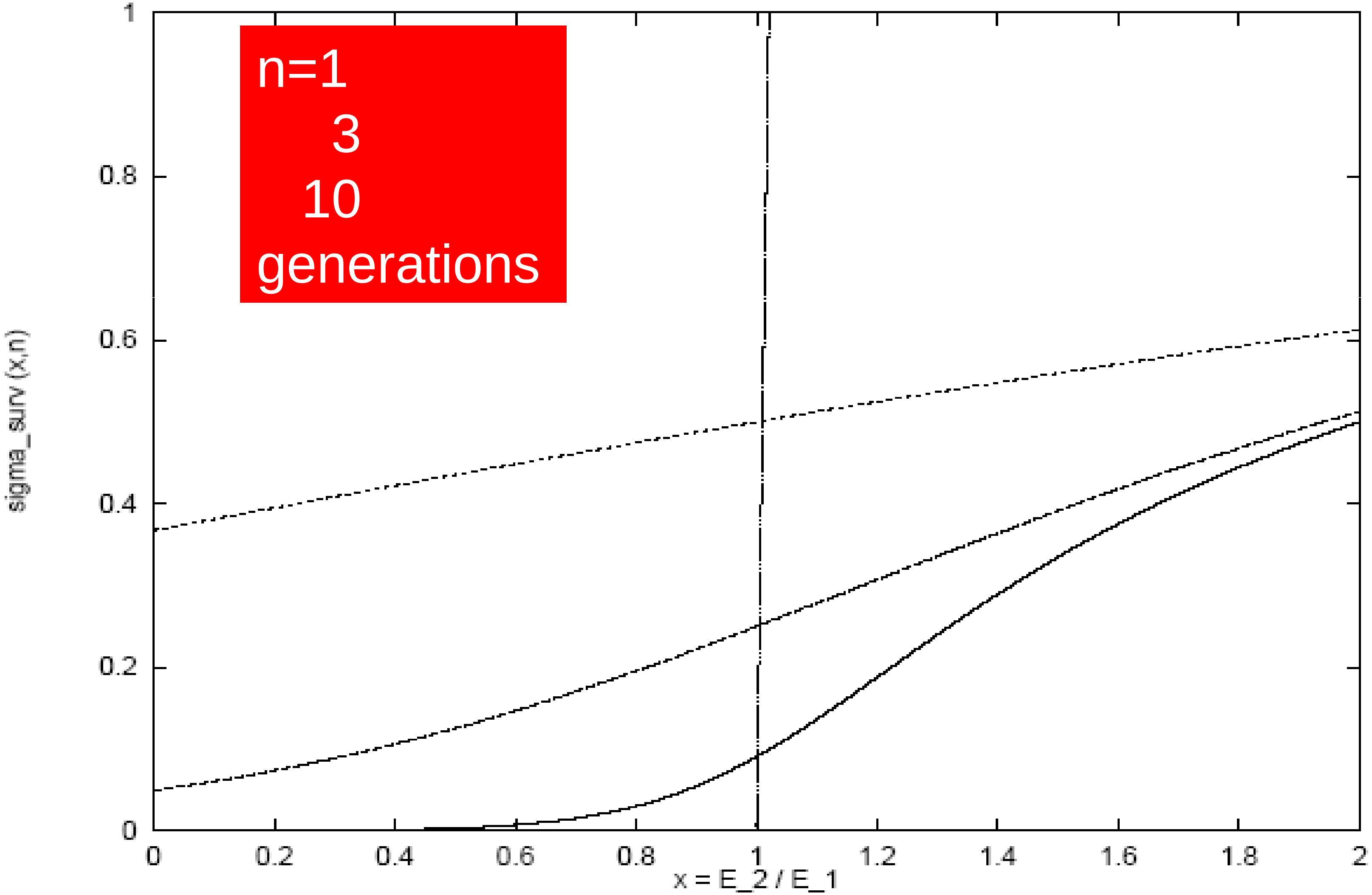
$$\sigma_2 = \begin{cases} 0 & \text{for} \quad E_2 < E_1 \\ 1 - \left(\frac{E_1}{E_2}\right)^{N_2(0)} & \text{for} \quad E_2 > E_1 \end{cases} \qquad (24)$$

with $N_2(0) = N_2(t = 0)$ the initial state of the system. Is $N_2(0)$ the number of users at time $t = 0$ of the technology 2, then σ_2 is the probability, that for $t \to \infty$ $N_2 = N$ users change to technology 2. σ_1 is the probability that for $t \to \infty$ $N_2 = 0$ (i.e., $N_1 = N$) the system returns to species 1, this means the new species has not survived. In general, σ_i is the survival probability of

Prob. of survival infinite generations in dep. on pop. size N=3,5,100 + determ. result



Prob of survival n=10,3,1 generations (from below) and determin. result as function of relative advantage (t-large)



Traditional conclusions get vague: Bad/Neutral/Better

- Deleterious?
- neutral ??????
- Advantageous?

- Neutrality gets a new dynamic meaning (depending on N and n) !!!

Nonlinear rates = hyperzyclic techn nets (selfacceleration)

- DETERMINISTIC picture:
- growth is hyperbolic ! (singular at a finite time)
- Result depends not only on advantage but also on initial conditions !
- The (untercritical) NEW has no Chance ! (once-forever selection)
- Ex: modern Infotec (Windows,Google,..)

Simplest model: lin+quad rate terms

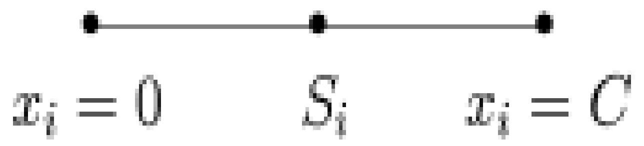
$$\dot{x}_i = E_i x_i + b_i\, x_i^2 \;-\; \varphi(t)\, x_i\,; \quad i \;=\; 1,2 \qquad (29)$$

The function $\varphi(t)$ follows from

$$x_1 \;+\; x_2 \;=\; \frac{N}{V} \;=\; C \;=\; \text{const.}$$

Equations of the same form have been derived for so-called hypercyclic system to describe the evolution of macromolecules [0]. Their behaviour is very well understood. As a result of the quadratic terms in the growth rates the phase space is split into two regions separated by a separatrix S_i. In the simplest case $E_i = 0$ we have: A certain species i only can win ($x_i = C$ for $t \to \infty$), if

Separatrix



Stochastic problems with nonlinear rates: New results !

For the general case of linear and quadratic rates the survival probability for a new species $\sigma_{N_2(0),N}$ can be calculated:

$$\sigma_{N_2(0),N} = \frac{1 + \sum_{j=1}^{N_2(0)-1} \prod_{i=1}^{j} \frac{E_1 + b_1 \frac{N-i}{V}}{E_2 + b_2 \frac{i}{V}}}{1 + \sum_{j=1}^{N-1} \prod_{i=1}^{j} \frac{E_1 + b_1 \frac{N-i}{V}}{E_2 + b_2 \frac{i}{V}}} \qquad (32)$$

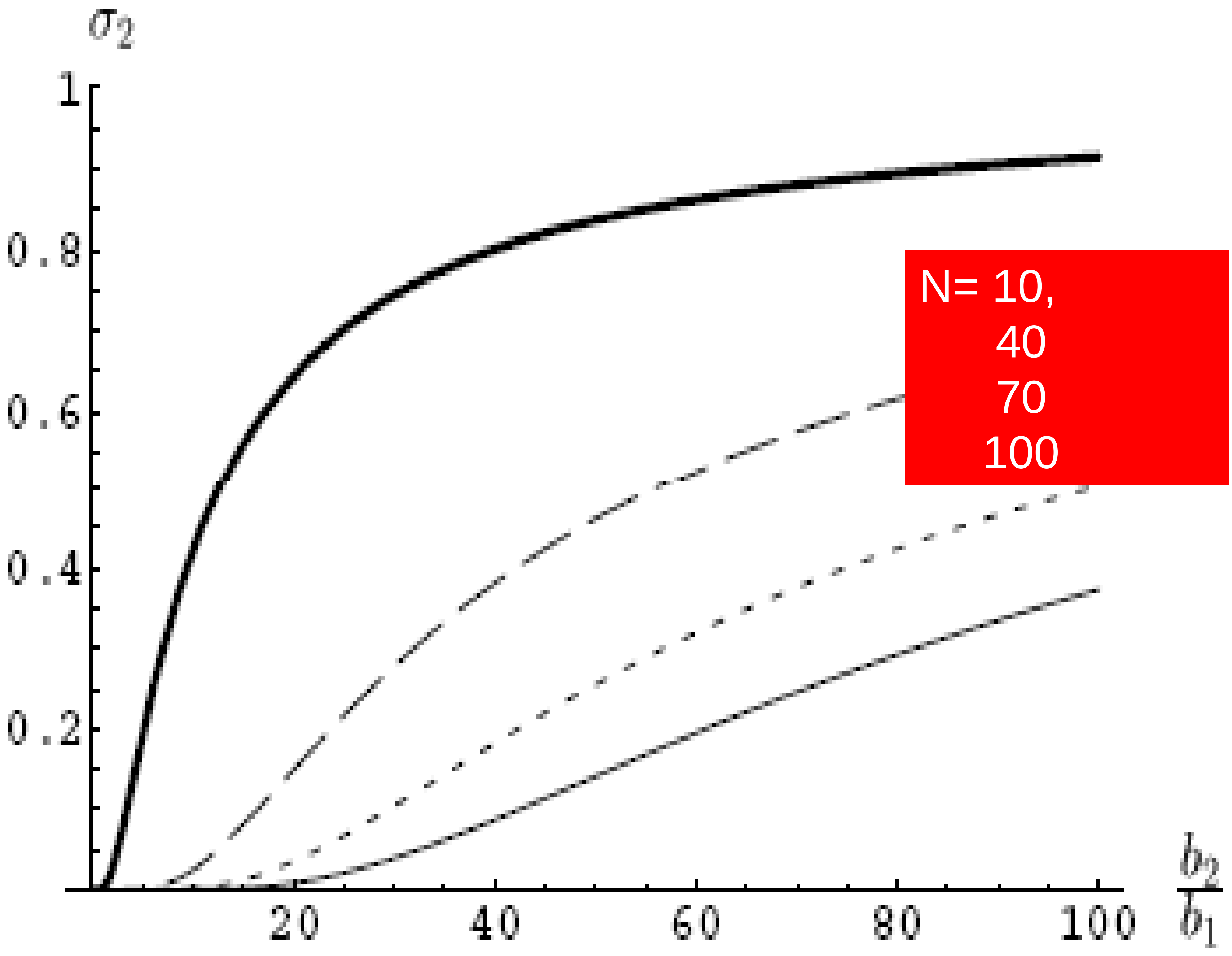
Simple for N_2 (0) = 1

1 in numerator remains

Special case of quadr growth b_i x_i^2

A special case of this formula is obtained for the case of purely quadratic growth $E_i = 0$. Then we get for the case $N_2(0) = 1$ (only one individual of soecies 2 occurs at $t = 0$):

$$\sigma_2 = \frac{1}{\left(1 + \frac{b_1}{b_2}\right)^{N-1}} \tag{33}$$



σ2
1
0.8
0.6
0.4
0.2
20
40
60
80
100
b2/b1
N= 10,
40
70
100

Summary of stochastic effects:

Das Neue (auch HC) hat eine Chance (survival prob > 0)

$$\sigma_{N_2(0),N} = \frac{1 + \sum_{j=1}^{N_2(0)-1} \prod_{i=1}^{j} \frac{E_1 + b_1 \frac{N-i}{V}}{E_2 + b_2 \frac{i}{V}}}{1 + \sum_{j=1}^{N-1} \prod_{i=1}^{j} \frac{E_1 + b_1 \frac{N-i}{V}}{E_2 + b_2 \frac{i}{V}}}$$

Hypercyclic nets of technol are qualitat different from linear nets!

- Deterministic picture: If a separatrix exists, the NEW has no chance at all.
- Exception: the NEW gets support, to cross the separatrix

- Stochastic picture: New hypertechns with better rates have a good chance.
- However this is true only for small niches

A few references: discrete m.

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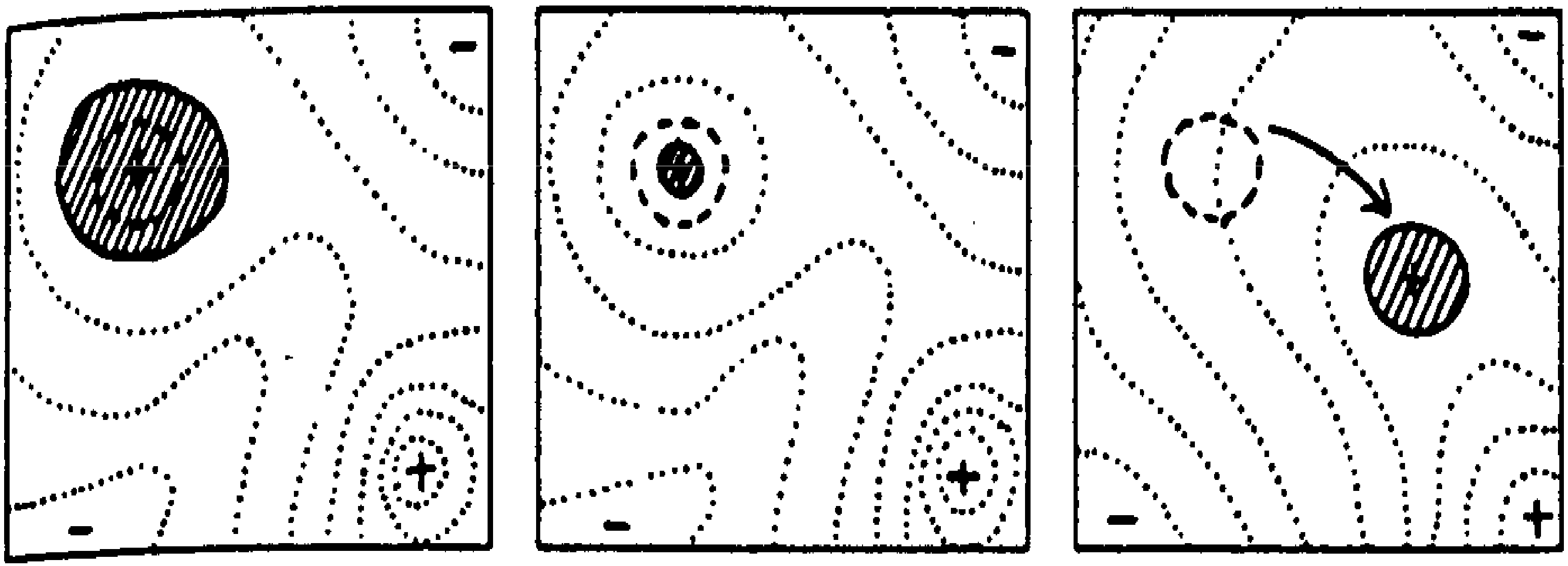
3. Brownian agents modelling transitions to new technologies

- Idea: Describe Techn by a set of cont Parameters: Heigth, weigth, size, power, techn data , ...,
- LANDSCAPE
- Space of cont. Charakteristika (Metcalfe, Saviotti seit 1984)
- Scharnhorst: G_O_E_THE (geometrical oriented Evolution theory)

Idee aus der Biologie: Wright Fitness landscape
Evolution as Optimization Process

Adaptive Landscape/Fitness Landscape



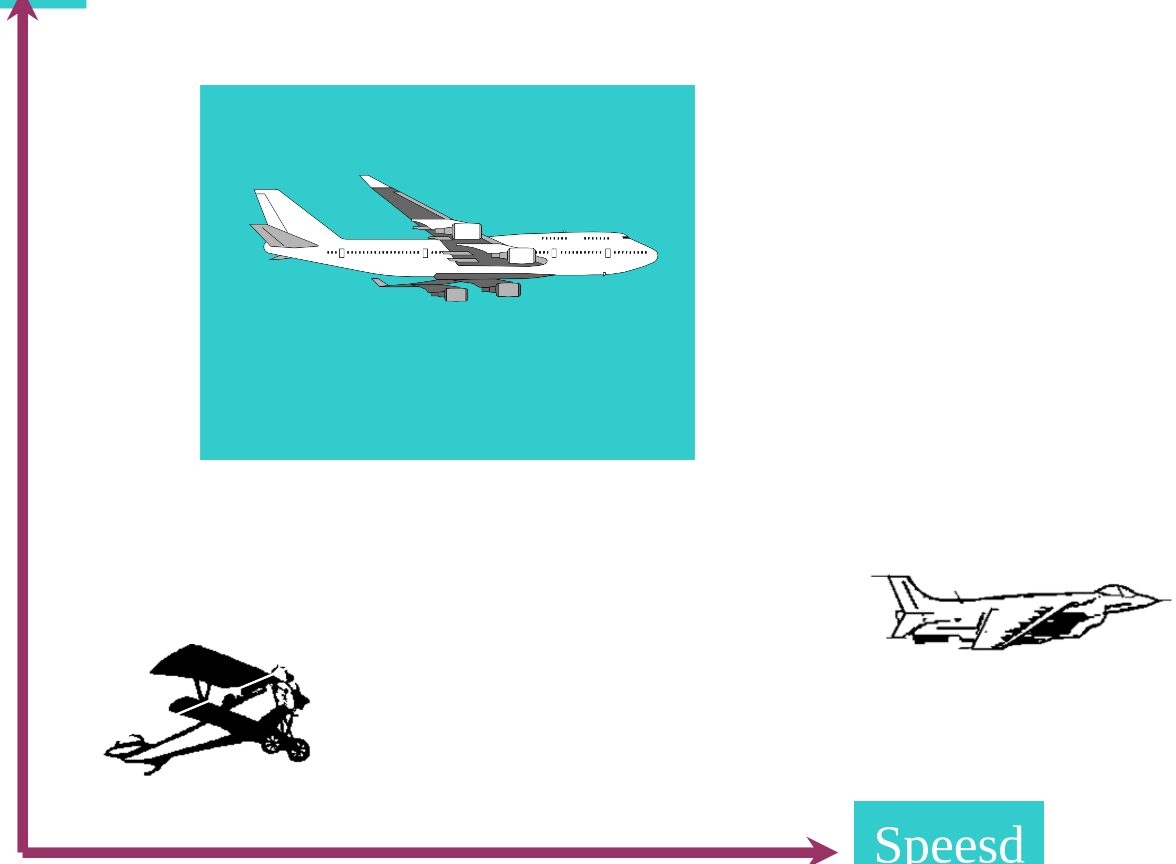
A. Increased Mutation or reduced Selection

B. Increased Selection or reduced Mutation

C. Qualitative Change of Environment

Basics

Characteristics Space

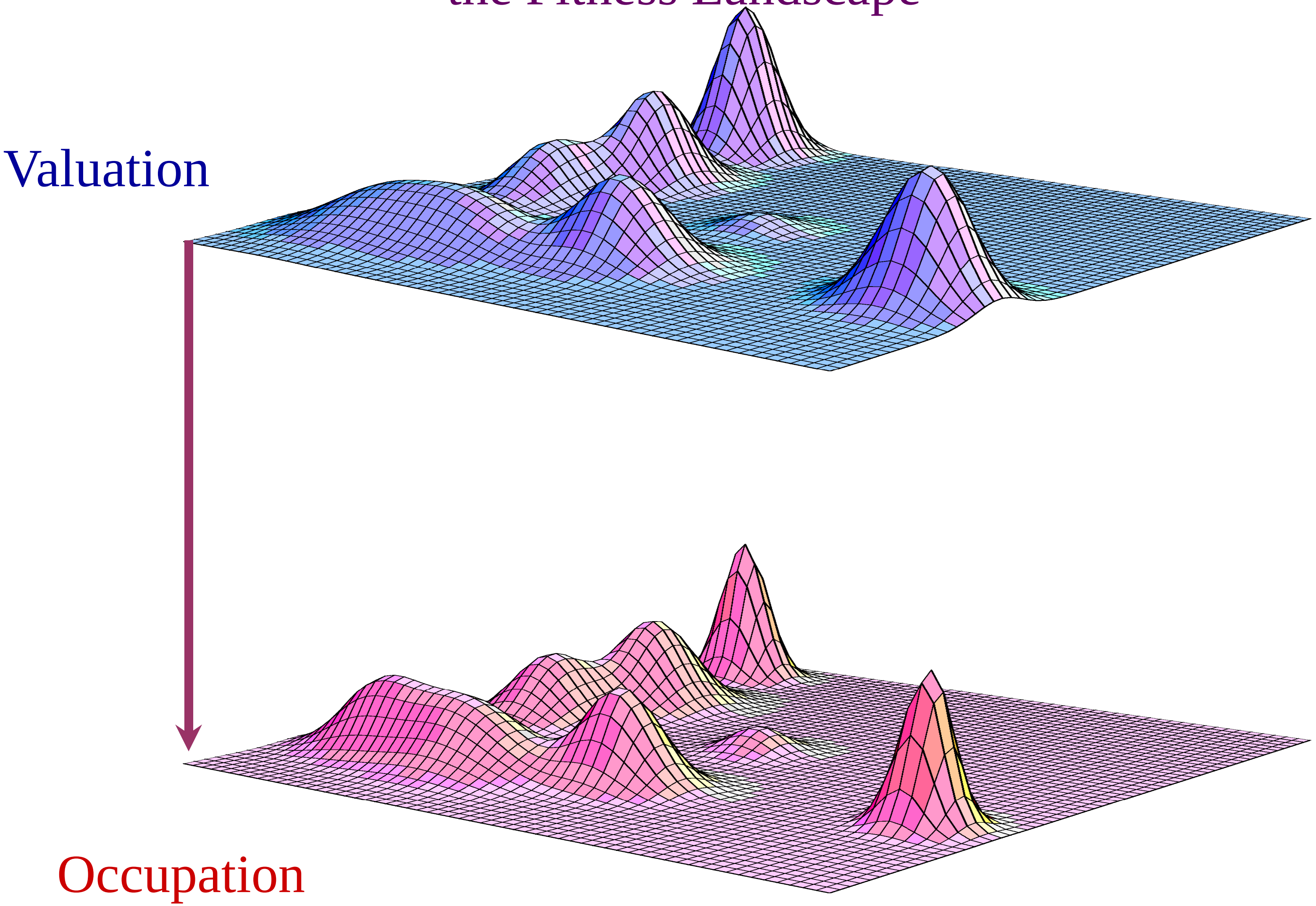


Technological Evolution:

Characteristics Space of Output Indicators

Metcalfe, Saviotti 1984

The Occupation Landscape Changes According to the Fitness Landscape



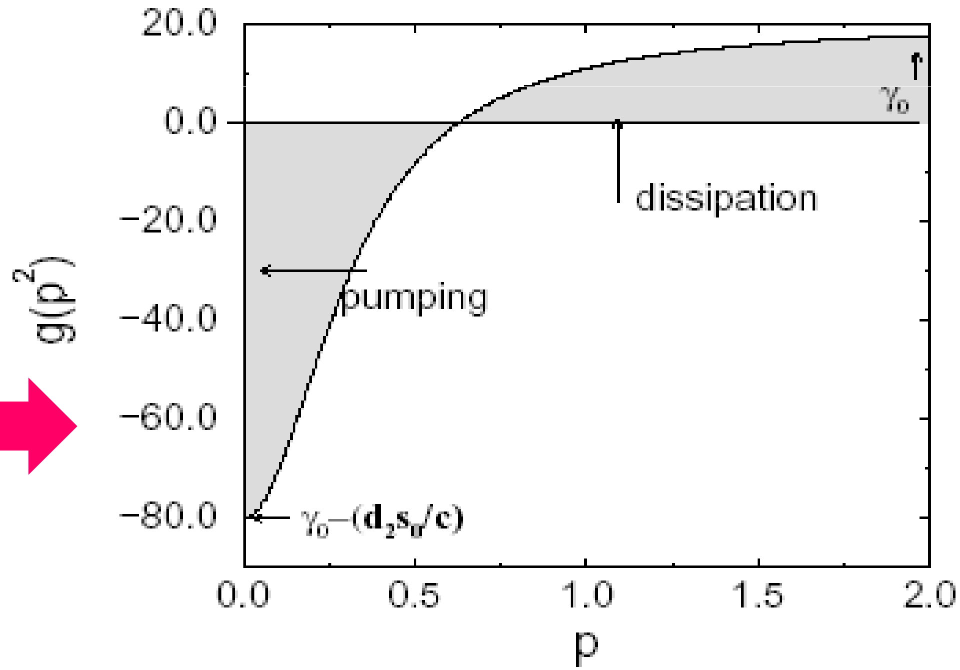
Evolutionary theory (Eigen/Schuster): d.e. corr to overdamped Langevin-eq. or diffusion eq for conc.

$$\gamma_0 \frac{d}{dt}\mathrm{x} + \frac{1}{\mathrm{m}}\frac{dU}{dr} = +\sqrt{2\mathrm{D}} \cdot \xi(t); \quad v = \frac{d}{dt}\mathrm{x}$$

In the ABM - model we consider the velocities as independent coordinates, consider inertia and driving !!!

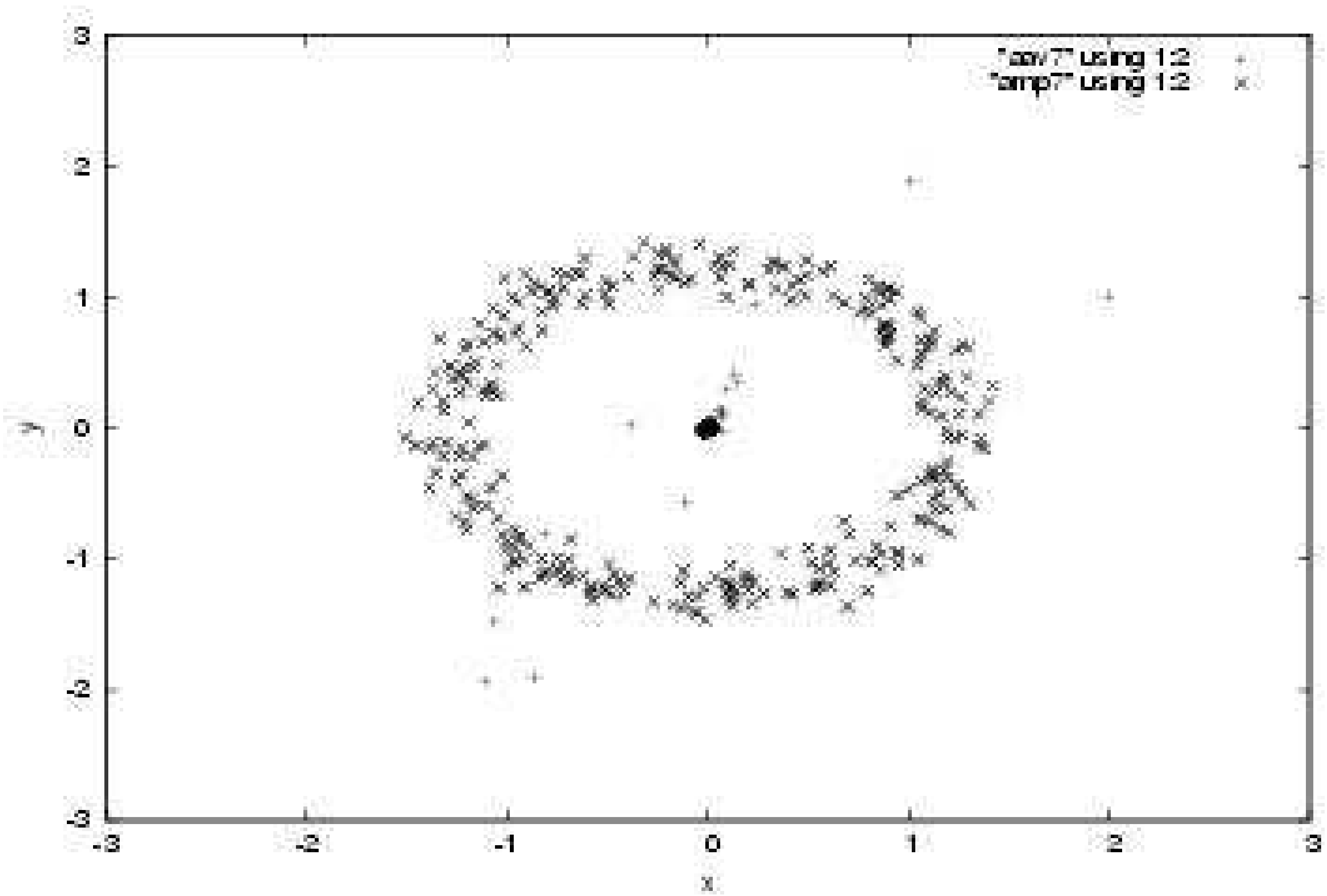
$$\frac{d}{dt}\mathrm{v} + \frac{1}{\mathrm{m}}\frac{dU}{dr} = -\gamma(v^2)\mathrm{v} + \sqrt{2\mathrm{D}} \cdot \xi(t)$$

Depot model - SET

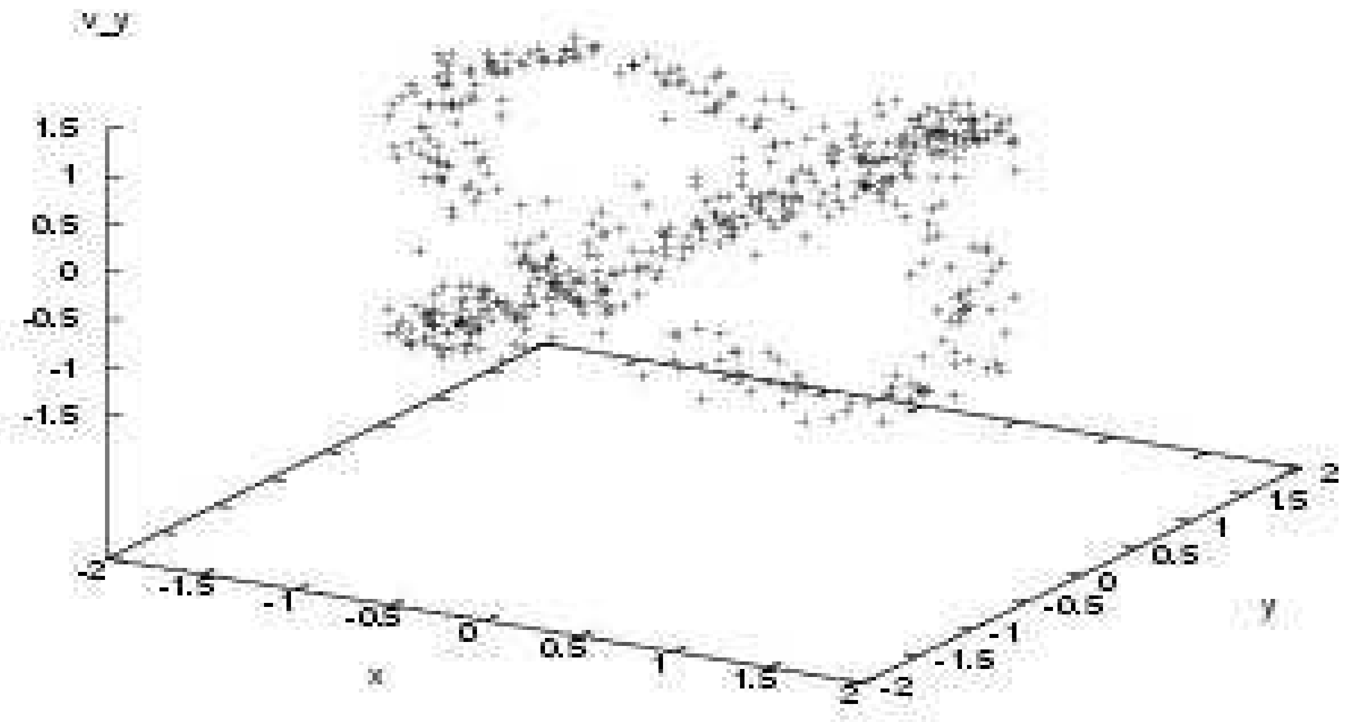


Active friction: Zero of the velocity $v_0^2 = \frac{d}{c}\mu; \quad \mu = \frac{qd}{c\gamma_0} - 1$

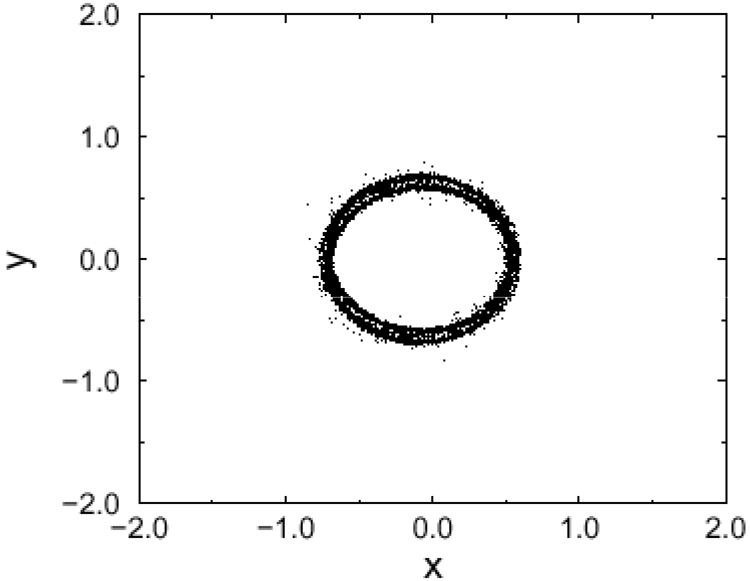
Dynamik von Techn, die linear zum Zentrum getrieben werden

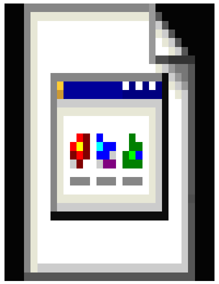


Rotationen (links/rechts): (limit cycles)



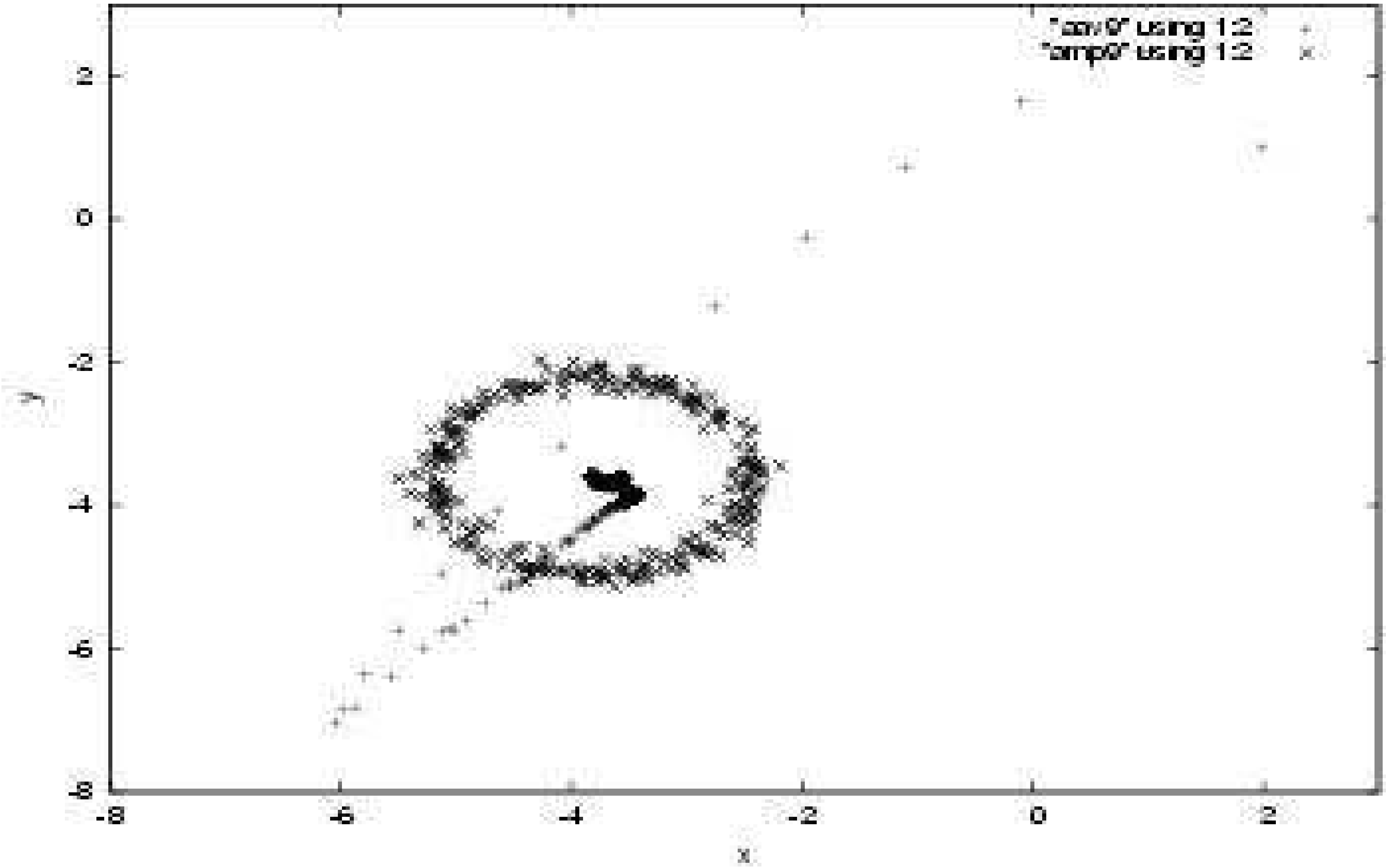
10000 aktive Teilchen um linear anziehendes Zentrum: Einschwingprozess !



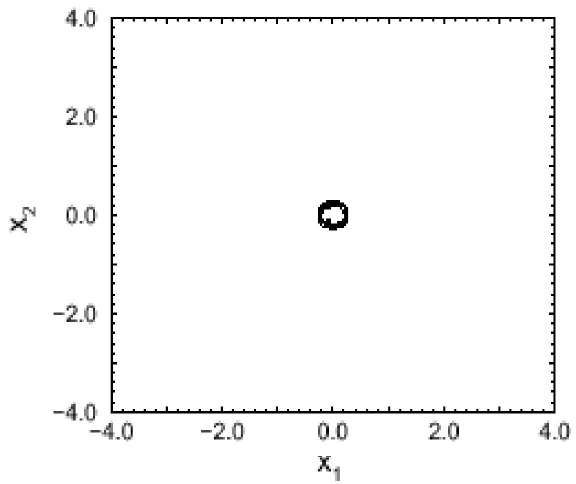


Swaest1.gif

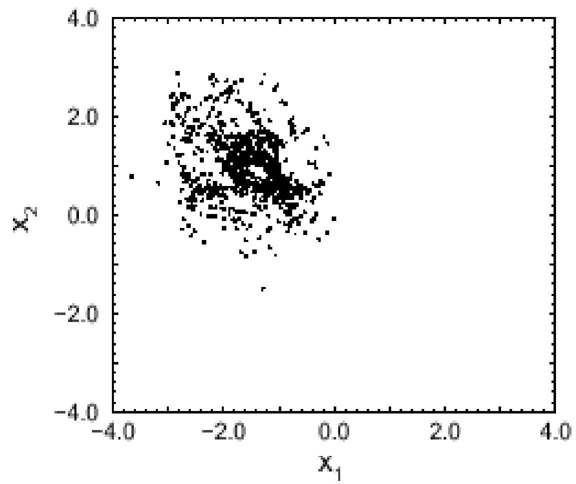
Rotations around a center



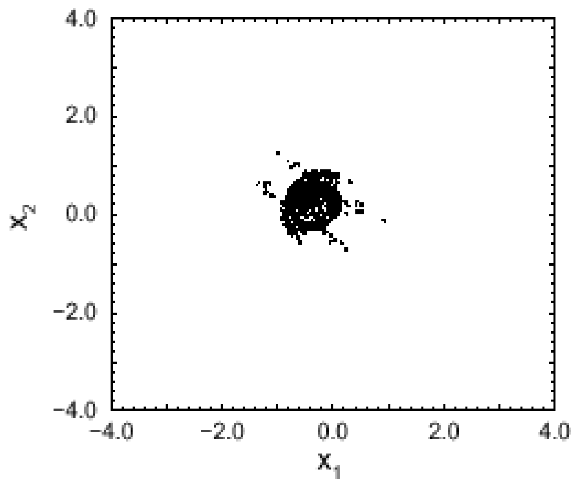
2000 Agenten mit paarweise linearer Anziehung



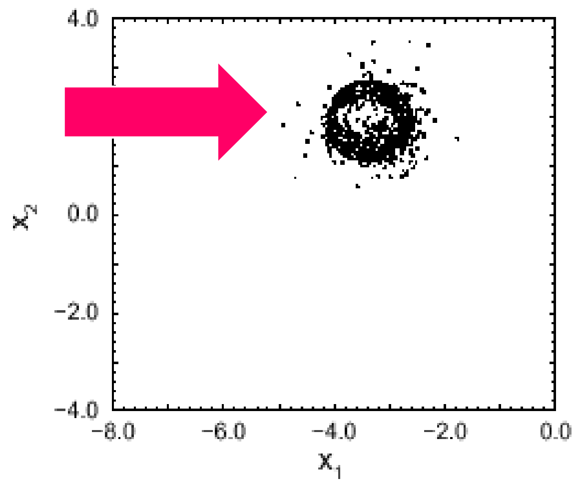
t=1



t=25



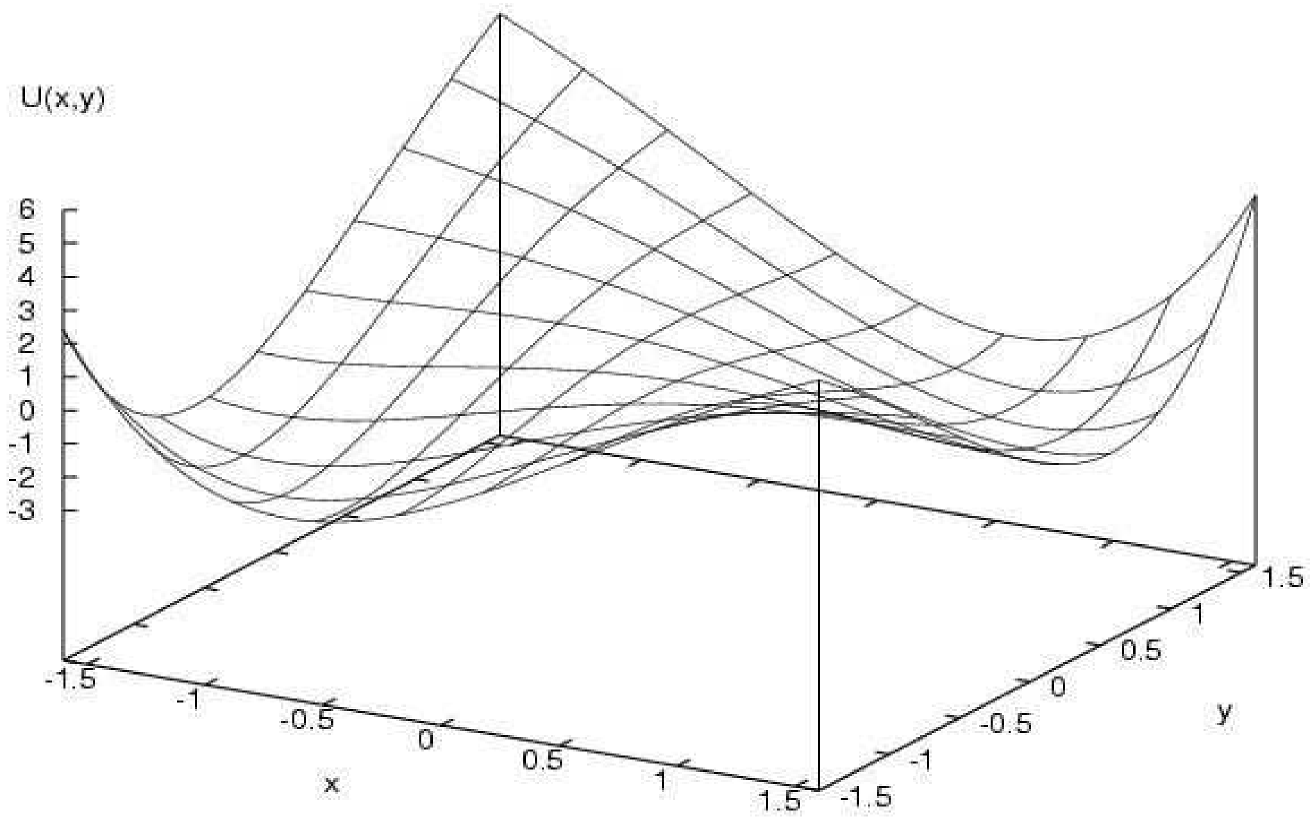
t=10



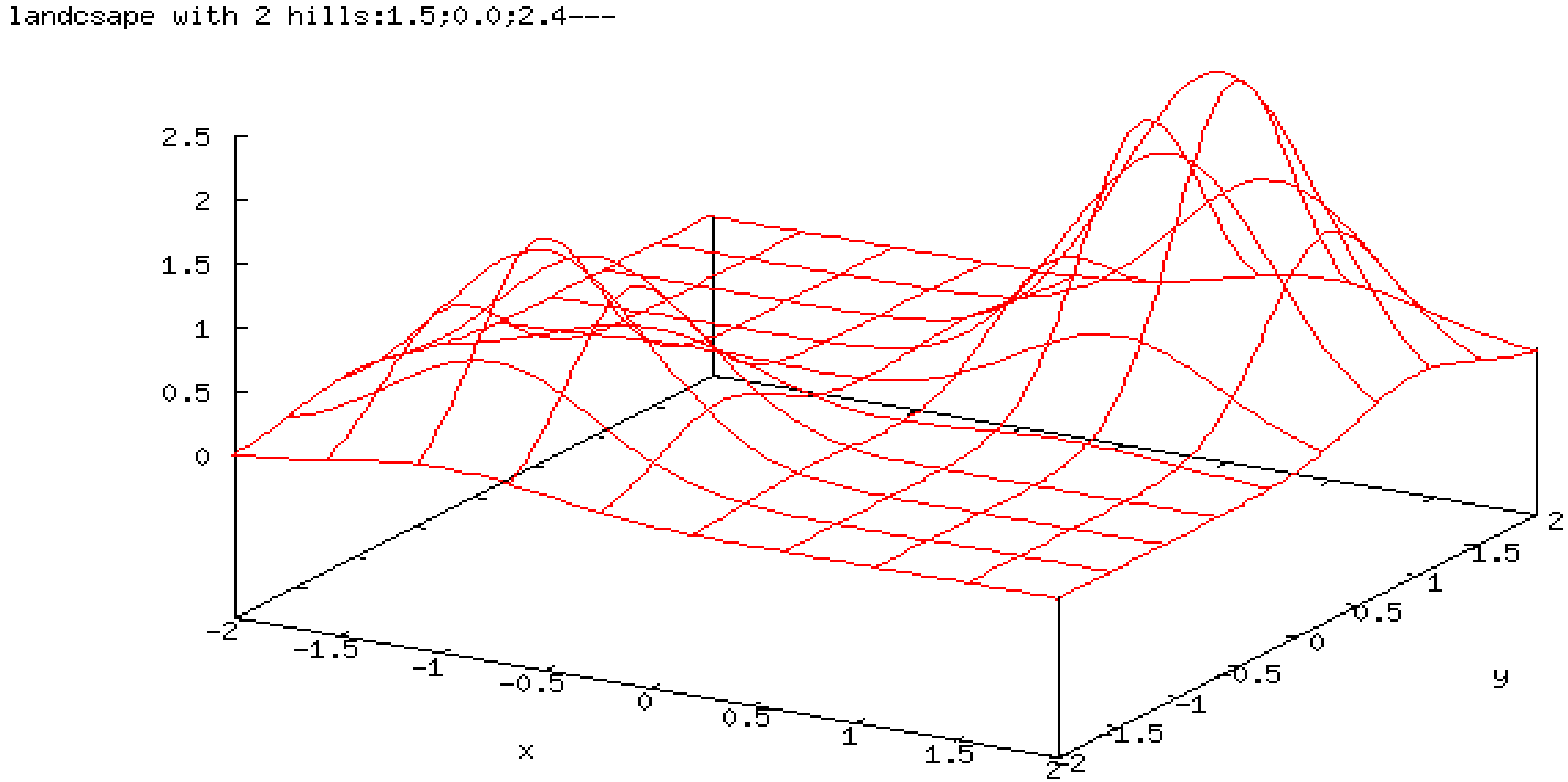
t=50

Swharm.gif

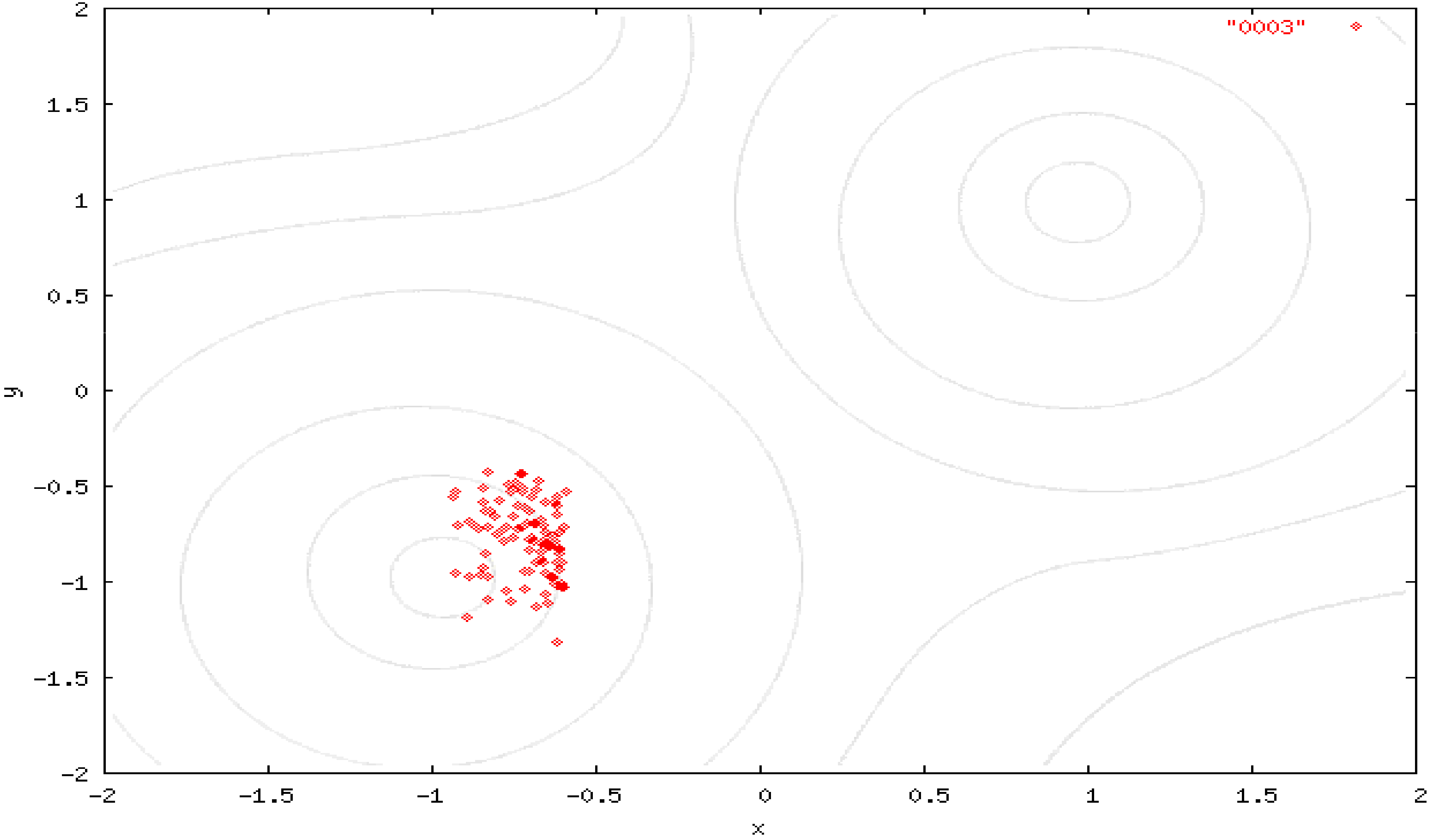
Transitions between two attractors (from good to better)



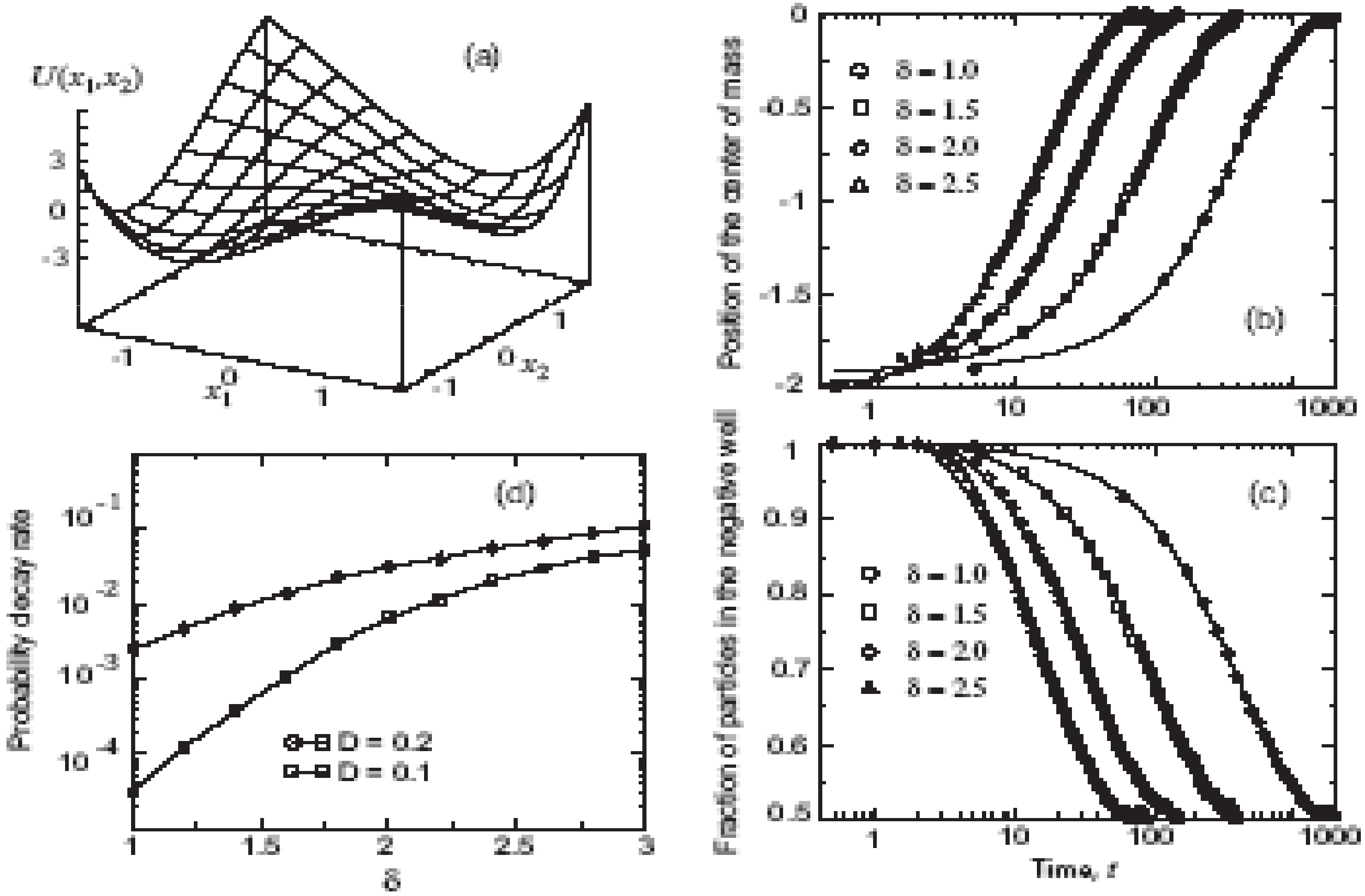
Landscape with 2 hills



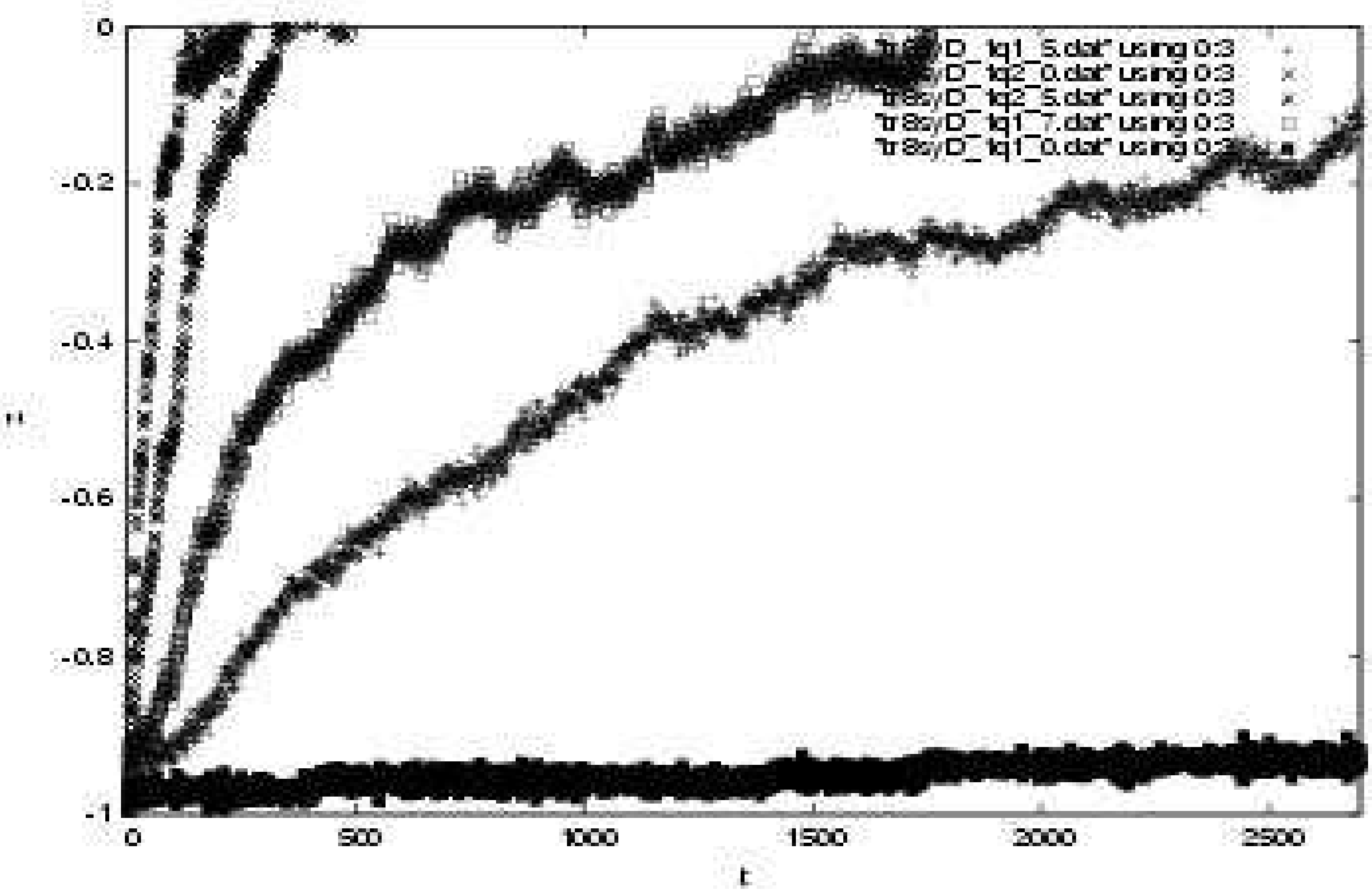
Simulation of the transition of agents



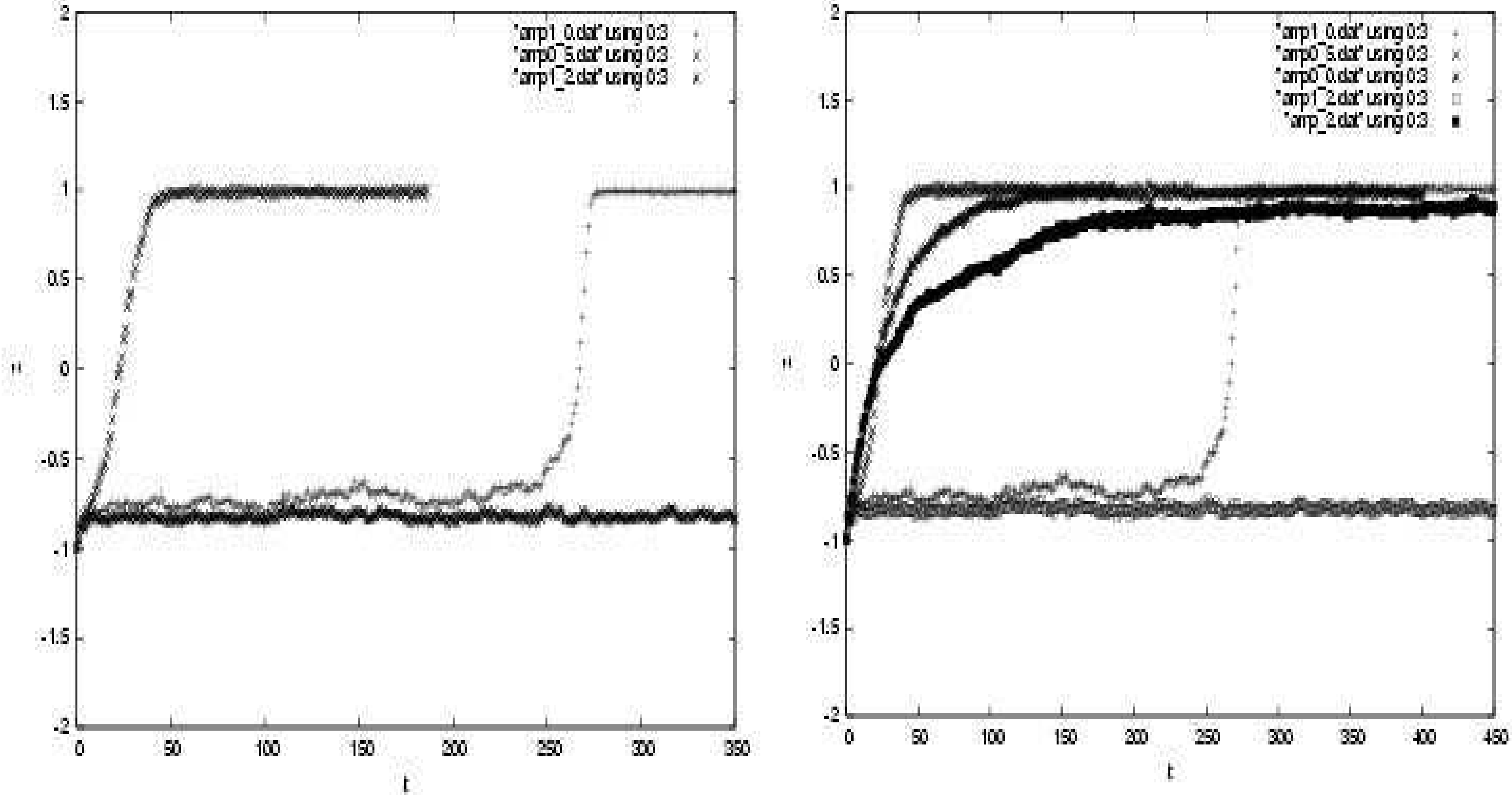
Statistical data, transition time etc dep on parameters



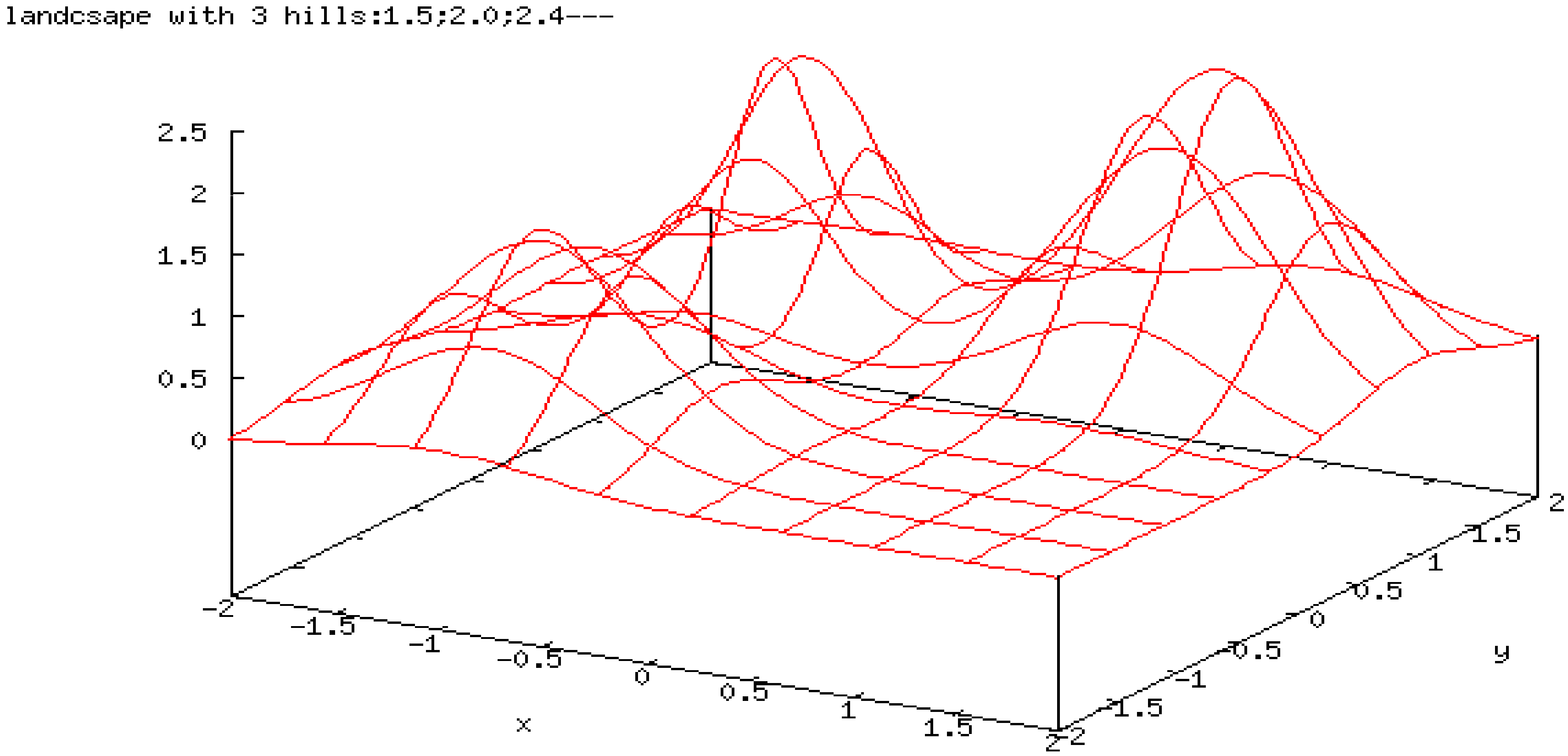
time of transition (well to well) for increasing strength of drive



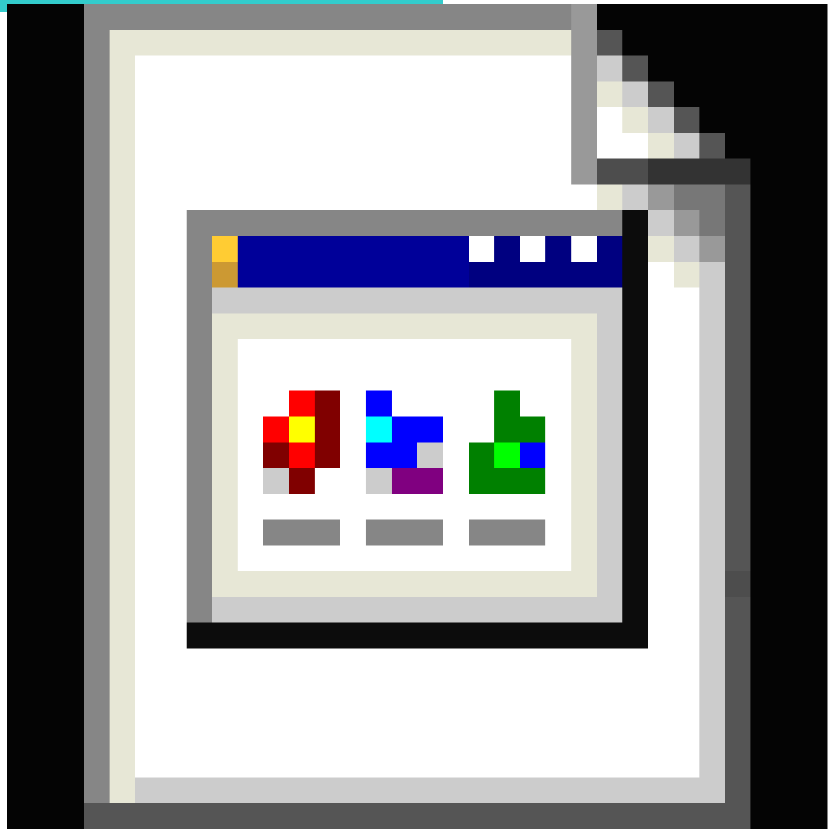
the effect of collective relative attraction



Wertelandschaft mit 3 Maxima



Übergänge zwischen 3 Werte Maxima



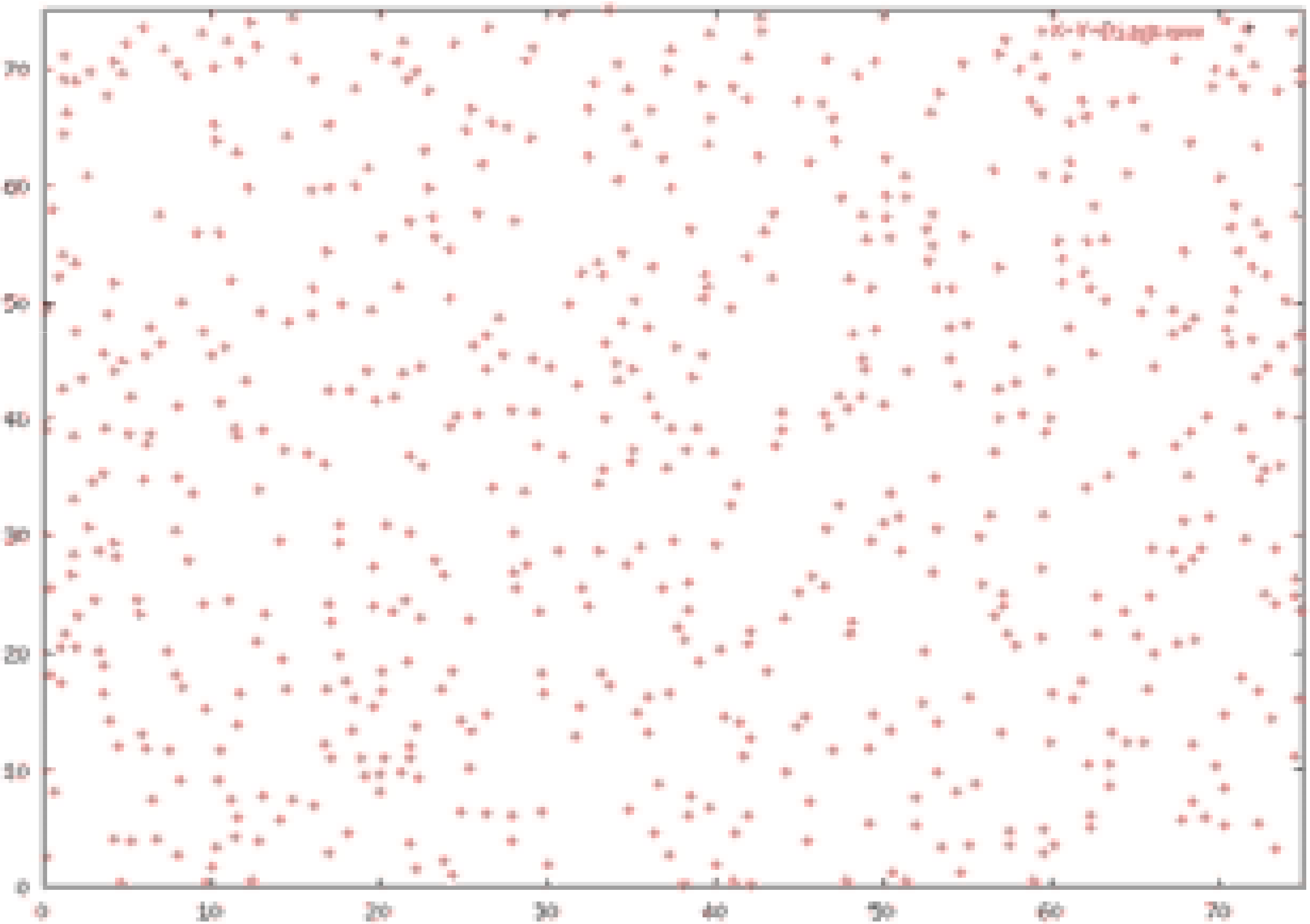
3max46.exe

Landscapes with many extrema

1. Ratchets (Saw tooth -Potentials)

2. Landscapes with randomly distr extrema

Evolution of networks of agents (illustration by Erdmann)



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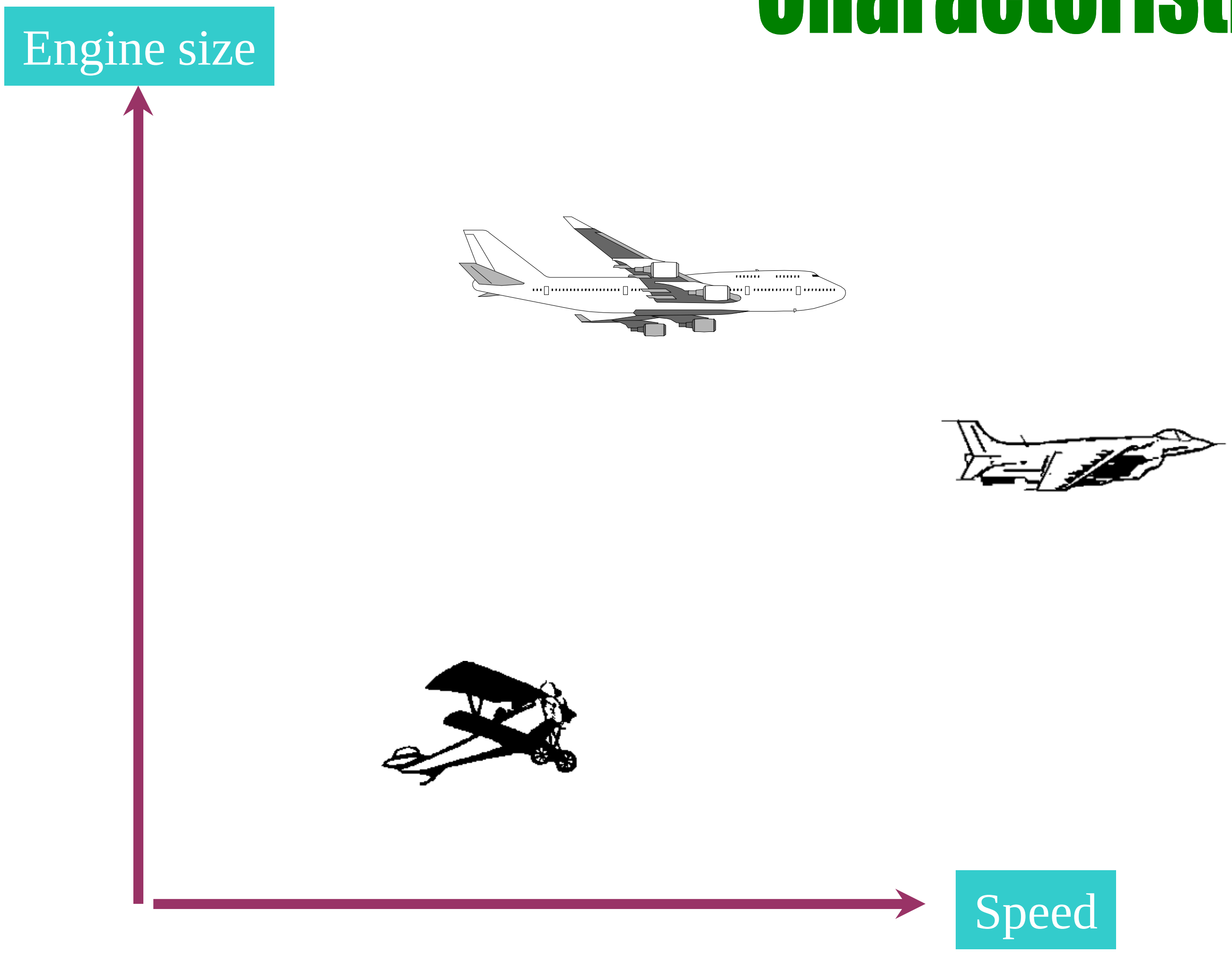
Conclusions

- Stochastic effects may be important for socio-economic processes:
- In the framework of linear rate theory, stochastic selection is quite neutral, to win the competition, the NEW needs big advantage !!!
- Hypercyclic systems can win in small niches,
- Complex transition/evolution processes may be described by dynamics on landscapes.
- Mathematical difficulties are relatively high !!!

Solve if possible

- Analytical solutions. This is possible only for a few examples as:
- The Fisher-Eigen-Schuster problem
- Survival probabilities
- Simulations by means of a fast computer with sufficient memory
- Formulate efficient algorithms
- Extrapolate and compare with analytical results

Characteristics Space



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